The `atan2()` function

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The function $tan^{-1}\frac{y}{x}$ has two input parameter, y and x

When $x, y < 0$, for example, (-3,-1) , $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1}{-1} = \tan^{-1}\frac{1}{3}$, it is same as the input $(+3, 1)$

Then the function can not differentiate $(\pm x, \pm y)$, the output is **not unique**

When tackling a problem that uniqueness of the solution is required (for example , inverse kinematics problem in robotics) , then a better function `atan2()`is used, the mathematical definition of atan2() is

$$\texttt{atan2(y,x)} = \begin{cases} \tan^{-1}\frac{y}{x} & x > 0 \\ \left(\tan^{-1}\frac{y}{x}\right) + \pi & x < 0\,, y \geq 0 \\ \left(\tan^{-1}\frac{y}{x}\right) - \pi & x < 0\,, y < 0 \\ +\frac{\pi}{2} & x = 0\,, y > 0 \\ -\frac{\pi}{2} & x = 0\,, y < 0 \\ \text{Undefined} & x = y = 0 \end{cases}$$

The `atan2(y,x)` also equals to

$$\texttt{atan2(y,x)}=\frac{2y}{\sqrt{x^2+y^2}+x}$$

The code implementation in C++ style

```
long double atan2( double y, double x){
     if ( x > 0.0 ){
         return atan(y/x);
     }
     else if ( x < 0.0 ){
             if ( y > 0.0 ){
                return ( atan(y/x) + PI );
             }  else
                return ( atan(y/x) - PI );
          }
     // x = 0.0
     if ( y > 0.0 ){
        return (PI/2) ;
     } else if ( y < 0.0 ){
                 return (-PI/2);
     } else
           return NaN; // Not a Number
}
```

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